APPENDIX: TUNED MODELS OF PEER ASSESSMENT IN MOOCS

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In this document, we describe the inference/learning procedures used in our paper.

1. GIBBS SAMPLING FOR MODEL \mathbf{PG}_1

Model \mathbf{PG}_1 is given as follows:

(Reliability) $\tau_v \sim \mathcal{G}(\alpha_0, \beta_0)$ for every grader v, (Bias) $b_v \sim \mathcal{N}(0, 1/\eta_0)$ for every grader v, (True score) $s_u \sim \mathcal{N}(\mu_0, 1/\gamma_0)$ for every user u, and (Observed score) $z_u^v \sim \mathcal{N}(s_u + b_v, 1/\tau_v)$,

for every observed peer grade.

The joint posterior distribution is:

$$\begin{split} P(Z|\{s_u\}_{u\in U},\{b_v\}_{v\in G},\{\tau_v\}_{v\in G}) \\ &= \prod_u P(s_u|\mu_0,\gamma_0) \cdot \prod_v P(b_v|\eta_0) \cdot P(\tau_v|\alpha_0,\beta_0) \prod_{z_u^v} P(z_u^v|s_u,b_v,\tau_v). \end{split}$$

The pseudocode for Gibbs sampling from Model \mathbf{PG}_1 is:

• Generate an initial assignment to all non-observed variables, s_u , τ_v , b_v for all true grades, grader reliabilities and grader biases. • For t = 1, ..., T: - For each user score s_{u_i} : * Sample $s \sim \mathcal{N}\left(s; \frac{\gamma_0}{\gamma_0 + \sum_{v:v \to u_i} \tau_v} \mu_0 + \frac{\sum_{v:v \to u_i} \tau_v(z_{u_i}^v + b_v)}{\gamma_0 + \sum_{v:v \to u_i} \tau_v}, \gamma_0 + \sum_{v:v \to u_i} \tau_v\right)$ * $s_{u_i} \leftarrow s$ - For each grader reliability τ_{v_i} : * Sample $\tau \sim \mathcal{G}\left(\tau; \alpha_0 + \frac{n_{v_i}}{2}, \beta_0 + \frac{1}{2}\sum_{u:u \to v_i} (z_u^{v_i} - (s_u + b_{v_i}))^2\right)$ * $\tau_{v_i} \leftarrow \tau$ - For each grader bias b_{v_i} : * Sample $b \sim \mathcal{N}\left(b; \frac{\sum_{u:u \to v_i} \tau_{v_i}(z_u^{v_i} - s_u)}{\eta + n_{v_i} \tau_{v_i}}, \eta + n_{v_i} \tau_{v_i}\right)$ * $b_{v_i} \leftarrow b$ - Save sample $\zeta^{(t)} \leftarrow (\{s_u\}_{u \in U}, \{\tau_v\}_{v \in U}, \{b_v\}_{v \in U})$ • Return samples from $\zeta^{(B)}, \zeta^{(B+1)}, \ldots, \zeta^{(T)}$ for some large enough number B. **Derivation of updates.** We examine the problems of sampling s_u and τ_v separately. Consider now a fixed user u_i . We derive the sampling step for s_u as follows:

$$s \sim P(s_{u_i}|MB(s_{u_i})),$$

$$\propto P(s_{u_i}|\mu_0, \gamma_0) \cdot \prod_{v:v \to u_i} P(z_{u_i}^v|s_u, b_v, \tau_v),$$

$$\propto \exp\left(-\frac{1}{2}\gamma_0(s_{u_i-\mu_0})^2 + \sum_{v:v \to u_i} \left(-\frac{1}{2}\tau_v \left(z_{u_i}^v - (s_{u_i}+b_v)\right)^2\right)\right),$$

$$\propto \exp\left(-\frac{1}{2}\left[\gamma_0(s_{u_i}-\mu_0)^2 + \sum_{v:v \to u_i} \tau_v \left(z_{u_i}^v - (s_{u_i}+b_v)\right)^2\right]\right).$$

The expression inside the exponent is quadratic — we thus complete the square, obtaining:

Therefore the sampling distribution is Gaussian:

$$s \sim \mathcal{N}\left(s \; ; \; \frac{\gamma_0}{\gamma_0 \sum_{v:v \to u_i} \tau_v} \mu_0 + \frac{\sum_{v:v \to u_i} \tau_v(z_{u_i}^v - b_v)}{\gamma_0 + \sum_{v:v \to u_i} \tau_v}, \gamma_0 + \sum_{v:v \to u_i} \tau_v\right)$$

Now consider a fixed user v_i . We derive the sampling step for grader reliability τ_v as follows:

$$\begin{aligned} \tau &\sim P(\tau_{v_i} | \operatorname{MB}(\tau_{v_i})), \\ &\propto P(\tau_{v_i} | \alpha_0, \beta_0) \cdot \prod_{u: u \to v_i} P(z_u^{v_i} | s_u, \tau_{v_i}, b_{v_i}), \\ &\propto \tau_{v_i}^{\alpha_0 - 1} \exp\left(-\beta_0 \tau_{v_i} + \sum_{u: u \to v_i} \frac{1}{2} \left(\log \tau_{v_i} - \log 2\pi - \tau_{v_i} \left(z_u^{v_i} - (s_u + b_{v_i})\right)^2\right)\right), \\ &\propto \tau_{v_i}^{\alpha_0 + \frac{nv_i}{2} - 1} \exp\left(-\left[\beta_0 + \frac{1}{2} \sum_{u: u \to v_i} \left(z_u^{v_i} - (s_u + b_{v_i})\right)^2\right] \tau_{v_i}\right). \end{aligned}$$

From this, we can recognize the sampling distribution to be Gamma with:

$$\tau \sim \mathcal{G}\left(\tau \; ; \; \alpha_0 + \frac{n_{v_i}}{2} \; , \; \beta_0 + \frac{1}{2} \sum_{u:u \to v_i} (z_u^{v_i} - (s_u + b_{v_i}))^2 \right).$$

Finally we derive the sampling set for grader bias b_v as follows:

$$\begin{split} b &\sim P(b_{v_i} | \mathrm{MB}(b_{v_i})), \\ &\propto P(b_{v_i} | \eta_0) \cdot \prod_{u: u \to v_i} P(z_u^{v_i} | s_u, \tau_{v_i}, b_{v_i}), \\ &\propto \exp\left(-\frac{1}{2}\eta_0 b_{v_i}^2 - \frac{1}{2}\sum_{u: u \to v_i} \tau_{v_i} \left(z_u^{v_i} - (s_u + b_{v_i})\right)^2\right), \\ &\propto \exp\left(-\frac{1}{2}\left[\eta_0 b_{v_i}^2 + \sum_{u: u \to v_i} \tau_{v_i} \left((s_u + b_{v_i})^2 - 2z_u^{v_i} (s_u + b_{v_i})\right)\right]\right). \end{split}$$

The expression inside square brackets is quadratic, again allowing us to complete-the-square as follows:

$$\eta b_{v_i}^2 + \sum_{u:u \to v_i} \tau_{v_i} \left((s_u + b_{v_i})^2 - 2z_u^{v_i} (s_u + b_{v_i}) \right)$$

= const. + $(\eta_0 + \sum_{u:u \to v_i} \tau_{v_i}) b_{v_i}^2 - 2 \left(\sum_{u:u \to v_i} \tau_{v_i} (z_u^{v_i} - s_u) \right) b_{v_i},$
= const. + $R \left(b_{v_i} - \frac{1}{R} \left(\sum_{u:u \to v_i} \tau_{v_i} (z_u^{v_i} - s_u) \right) \right)^2,$

where $R = \eta_0 + \sum_{u:u \to v_i} \tau_{v_i} = \eta_0 + n_{v_i} \tau_{v_i}$. The sampling distribution for *b* is thus Gaussian with:

$$b \sim \mathcal{N}\left(b \; ; \; \frac{\sum_{u:u \to v_i} \tau_{v_i}(z_u^{v_i} - s_u)}{\eta_0 + n_{v_i}\tau_{v_i}}, \; \eta + n_{v_i}\tau_{v_i}\right).$$

2. TEMPORAL COHERENCE (MODEL \mathbf{PG}_2)

Model \mathbf{PG}_2 looks almost identical to \mathbf{PG}_1 with the exception of the fact that a grader's bias depends on her bias at the last homework assignment.

$$\begin{split} \tau_v^{(T)} &\sim \mathcal{G}(\alpha_0, \beta_0) \text{ for every grader } v, \\ b_v^{(T)} &\sim \mathcal{N}(b_v^{(T-1)}, 1/\omega_0) \text{ for every grader } v, \\ s_u^{(T)} &\sim \mathcal{N}(\mu_0, 1/\gamma_0) \text{ for every user } u, \text{ and} \\ z_u^{v,(T)} &\sim \mathcal{N}(s_u^{(T)} + b_v^{(T)}, 1/\tau_v^{(T)}), \\ & \text{ for every observed peer grade.} \end{split}$$

Since we handle assignments in an online fashion, we do not consider the possibility of using grades from Assignment T to retroactively go back and modify earlier grades. Due to the Markov nature of the model for bias in Model \mathbf{PG}_2 , inference at each timeslice (i.e. each homework assignment) is the same as that of Model \mathbf{PG}_1 with the exception that instead of using the same bias for all graders, each grader now has his own prior over bias.

3. GIBBS SAMPLING FOR MODEL \mathbf{PG}_3

Model \mathbf{PG}_3 is given as follows:

$$b_v \sim \mathcal{N}(0, 1/\eta_0)$$
 for every grader v ,
 $s_u \sim \mathcal{N}(\mu_0, 1/\gamma_0)$ for every user u , and
 $z_u^v \sim \mathcal{N}\left(s_u + b_v, \frac{1}{f_{\theta}(s_v)}\right)$,

for every observed peer grade,

where $f_{\theta}(s) \equiv \theta_1 \cdot s + \theta_0$. **PG**₃ is the only model that we cannot Gibbs sample in closed form. The joint probability distribution is written as:

$$P(Z|\{s_u\}_{u\in U},\{b_v\}_{v\in G},\{\tau_v\}_{v\in G})$$

= $\prod_u P(s_u|\mu_0,\gamma_0) \cdot \prod_v P(b_v|\eta_0) \prod_{z_u^v} P(z_u^v|s_u,s_v,b_v)$

Derivation of updates. Again we look at the cases of sampling s_u and b_v separately. Consider now a fixed user u_i . We derive the sampling step for s_u as follows:

$$s \sim P(s_{u_i}|MB(s_{u_i})),$$

$$\propto P(s_{u_i}|\mu_0, \gamma_0) \cdot \prod_{v:v \to u} P(z_u^v|s_u, s_v, b_v) \cdot \prod_{w:u \to w} P(z_u^v|s_w, s_u, b_v),$$

$$\propto \exp\left(-\frac{1}{2}\gamma_0(s_{u-\mu_0})^2\right)$$

$$\begin{split} &\cdot \prod_{v:v \to u} \exp\left(-\frac{1}{2}f_{\theta}(s_{v})(z_{u}^{v}-(s_{u}+b_{v}))^{2}\right) \\ &\cdot \prod_{w:u \to w} \sqrt{f_{\theta}(s_{u})} \exp\left(-\frac{1}{2}f_{\theta}(s_{v})[z_{w}^{u}-(s_{w}+b_{u})]^{2}\right), \\ &\propto \sqrt{f_{\theta}(s_{u})}^{k_{u}} \cdot \exp\left(-\frac{1}{2}\left[\gamma_{0}(s_{u}-\mu_{0})^{2}\right. \\ &\left.+\sum_{v:v \to u} f_{\theta}(s_{v})(z_{u}^{v}-(s_{u}+b_{v}))^{2}\right. \\ &\left.+\sum_{w:u \to w} f_{\theta}(s_{u})(z_{w}^{u}-(s_{w}+b_{u}))^{2}\right]\right), \\ &\left(\text{where } k_{u} \text{ is the number of people graded by } u\right) \\ &\propto f_{\theta}(s_{u})^{k_{u}/2} \cdot \exp\left(-\frac{1}{2}\left[R\left(s_{u}-\frac{y}{R}\right)^{2}\right]\right), \end{split}$$

where:

$$R = \gamma_0 + \sum_{v:v \to u} f_\theta(s_u), \text{ and}$$
$$y = \mu_0 \gamma_0 + \sum_{v:v \to u} f_\theta(s_v) (z_u^v - b_v) + \sum_{w:u \to w} \theta_1 (z_w^v - (s_w + b_v))^2.$$

Note that unlike its analog from Model \mathbf{PG}_1 , the sampling step for s_u in Model \mathbf{PG}_3 cannot be performed in closed form. In our experiments, we sample from a discretized approximation of the posterior distribution instead. We expect that a Laplace approximation would also be effective (and fast) for this problem as the posterior distributions typically "look" nearly Gaussian in practice.

We now turn to sampling the bias variables b_v . Note that there are no reliability variables to sample in Model \mathbf{PG}_3 .

$$\begin{split} b &\sim P(b_v | MB(b_v)), \\ &\propto P(b_v | \eta_0) \cdot \prod_{u:v \to u} P(z_u^v | s_u, s_v, b_v), \\ &\propto \exp\left(-\frac{1}{2} \left[\eta_0 b_v^2 + \sum_{u:v \to u} f_\theta(s_v) (z_u^v - (s_u + b_v))^2\right]\right), \\ &\propto \exp\left(-\frac{1}{2} \left[R(b_v - \frac{y}{R})^2\right]\right), \end{split}$$

$$R = \eta_0 + \sum_{u:v \to u} f_{\theta}(s_v), \text{ and}$$
$$y = \sum_{u:v \to u} f_{\theta}(s_u)(z_u^v - s_u).$$
4. RUBRICS

In the remaining pages, we provide a sample rubric that was used in peer grading for Stanford's HCI course.

Assignment 1: Needfinding

Category	Unsatisfactory	Bare minimum	Satisfactory effort & performance	Above & Beyond
Observations	0: No observations or completely irrelevant observations.	1: The student observed only one person and/or the student observed an activity unrelated to the brief.	3: The student observed only two people and/or the student observed an activity that would be better related to another brief.	5: The student observed three people in an activity clearly related to the brief.
Quality of Observations	0: No observations or completely irrelevant observations.	1: The student's observations did not demonstrate a breakdown or a design opportunity that was relevant.	3: The student's observations somewhat demonstrated a breakdown or design opportunity, but they were only somewhat relevant, were communicated poorly, or left major questions unanswered.	5: The student's observations clearly demonstrated a breakdown or design opportunity. The descriptions were well written, informative, and comprehensive.
Photos/Sketches	0: The student did not submit photos and captions related to each described observation, or submitted photos that were clearly not taken as part of the	1: The student submitted no relevant photos/sketches submitted that demonstrate a breakdown or design opportunity.	3: The student submitted two relevant photos/sketches that demonstrate a breakdown or design opportunity.	5: The student submitted three relevant photos/sketches that demonstrate a breakdown or design opportunity.

	actual observation (e.g. stock photos).			
Ideas for User Needs	0: The student did not come up with ideas for user needs or gave an irrelevant answer.	1: The student came up with 1 - 7 ideas for user needs.	3: The student came up with 8 - 14 ideas for user needs.	5: The student came up with 15+ ideas for user needs.
Quality of Ideas for User Needs	0: The student did not come up with ideas or gave an irrelevant answers.	1: Most of the ideas the student came up with were irrelevant, repeated, or obvious (didn't require observation).	3: Most of the student's ideas were insightful; Only a few seemed irrelevant, repeated, or obvious.	5: All of the ideas were insightful. Each idea could become the basis for a design project.

Assignment 2: Storyboarding

Category	Unsatisfactory	Bare minimum	Satisfactory effort and performance	Above & Beyond
Inspirations	0: The student did not come up with inspirations or gave an irrelevant answers.	1: The student found 1 - 2 inspirations.	3: The student found 3 - 4 inspirations.	5: The student found 5+ inspirations.
Quality of Inspirations	0: The student did not come up with inspirations or gave an	1: The inspirations had no explanations.	3: The inspirations were obvious (didn't require observation)	5: The inspirations were diverse and had insightful explanations.

	irrelevant answers.		and/or the explanations were vague and confusing (it would be hard to implement a solution based on them).	
Point of view	0: No point of view or irrelevant point of view.	1: The point of view does not express a problem or opportunity, does not create requirements for a solution, or does not relate to the design brief.	3: The point of view relates to the brief and the problem and solution are clearly stated, but the solution requirement is too general (any solution meets the requirement) or is too specific (allows for only one solution).	5: The point of view relates to the brief and the problem and solution are clearly stated; the requirement allows for focus without being too constraining.
Storyboard #1	0: The student did not submit a storyboard.	1: The storyboard is hard to follow or does not address the problems and solution requirements identified in the point of view	3: The storyboard reasonably addresses the problems identified in the point of view, but a reader may have lingering questions about the situation depicted.	5: The storyboard is easy to follow. Someone else could come up with a distinct prototype that would correspond with the point of view just from looking at the storyboard.
Storyboard #2	0: The student did not submit a second storyboard.	1: The storyboard is hard to follow and does not address the problems and solution requirements identified in	3: The storyboard reasonably addresses the problems identified in the point of view, but a reader may have lingering	5: The storyboard is easy to follow. Someone else could come up with a distinct prototype that would correspond with the point of view

	the point of view.	questions about the situation depicted and/or it does not diverge and represent a different solution from the first storyboard.	just from looking at the storyboard. It clearly diverges and represents a different solution from the first storyboard.
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Assignment 3: Prototyping

Category	Unsatisfactory	Bare minimum	Satisfactory effort and performance	Above & Beyond
Wireframe Prototype #1	0: No prototype or completely irrelevant prototype.	1: The prototype is not interactive or interactions are broken, and there is no defined purpose for many elements in the prototype.	3: The prototype is mostly complete, although there are some functions that are not yet interactive, some elements have no defined purpose, and it can be difficult to know how to use certain parts of the prototype.	5: The prototype has enough detail that the user can see all of the interactions, understand how they work, and a programmer could use the prototype to create a functional application with a defined flow.
Wireframe Prototype #2	0: No second prototype or completely irrelevant prototype.	1: The prototype is not interactive or interactions are broken, and there is no defined purpose for many elements in the prototype.	3: The prototype is mostly complete, although there are some functions that are not yet interactive, some elements have no defined purpose, and it can be difficult to know how to use certain parts of the prototype, and/or it	5: The prototype has enough detail that the user can see all of the interactions, understand how they work, and a programmer could use the prototype to create a functional application with a defined flow. The

			does not diverges from the first prototype and explore different interfaces implementing the same idea.	prototype clearly diverges from the first prototype and explores different interfaces implementing the same idea.
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Assignment 4: Start Building

Category	Unsatisfactory	Bare minimum	Satisfactory effort & performance	Above & Beyond
List of Changes	0: No changes or completely irrelevant changes.	1: The student only identified a few changes from the heuristic evaluation feedback and a large amount of feedback is ignored in the new prototype; the new prototype has some HE violations.	3: The student made many of the simpler suggestion changes, but some of the more complex or difficult issues were not addressed; the new prototype does not have any obvious HE violations.	5: The student made several insightful and specific changes based on the heuristic evaluation feedback. It is hard to find any HE violations at all in the new prototype.
Development Plan	0: No development plan or completely irrelevant plan.	1: The development plan does not address every step of development and does not create clear actionable tasks.	3: The development plan has several reasonable steps for development, but they are not clearly defined or do not cover all aspects of development.	5: The development plan has many distinct, logical steps that give a clear path for development.
Deadlines	0: No	1: The timeline	3: The timeline	5: The timeline

	deadlines or completely irrelevant deadlines.	seems haphazard and the deadlines are obviously impossible to follow.	is well- organized and mostly doable, although a few of the deadlines seem idealistic or unreasonable.	is well- organized, has feasible deadlines, and takes into account time for unforeseen issues.
Navigational Skeleton	0: No home screen or navigational skeleton.	1: Home screen has little content, and navigation does not work. OR, the student submitted a prototype URL, but the prototype wasn't viewable.	3: Home screen appears to have most of its content, and the major navigations are present.	5: Home screen and navigational skeleton are very thorough and well planned. The navigational skeleton gives a real feel for the flow of the application and is clearly thought through.

Assignment 5: Ready for Testing

Category	Unsatisfactory	Bare minimum	Satisfactory effort & performance	Above & Beyond
Interactive Prototype	0: No prototype or irrelevant prototype.	1: The prototype is not interactive, lacks many features, and has many bugs; the design does not work with the goal. OR, the student submitted a prototype	3: The prototype is mostly interactive, with only a few features missing and only one or two bugs; the design accomplishes the minimum requirements of	5: The prototype is completely interactive, reflects the feel of the final prototype, and is ready for user testing; the design accomplishes the entire goal.

		URL, but the prototype wasn't viewable.	the goal	
User Evaluation Plan: Completeness	0: No plan or irrelevant plan.	1: The evaluation plan exists, but is minimal, unclear, and is not well thought out.	3: The evaluation plan is mostly complete, but does not cover all questions about testing thoroughly (what is tested, what you want to learn, when, where, participants).	5: The evaluation plan is complete, answers all questions specifically, and shows a clear process for user testing.
User Evaluation Plan: Appropriateness	0: No plan or irrelevant plan.	1: The evaluation plan does not choose to evaluate aspects of the design related to the design goals.	3: The evaluation plan is designed to produce some useful data, but is not justified by the student (e.g. why are you doing what you are doing?- - why 6 participants? Why in a school? etc).	5: The evaluation plan is very clearly motivated or innovative in a way that will ensure rich and interesting data to address the design goals.
Development Goals	0: No goals met that were laid out on the development plan.	1: The student met a few of the goals laid out in the development plan.	3: The student met most, but not all, of the goals laid out in the development plan.	5: The student met all of the goals found in the development.

Assignment 6: User Testing

Category	Unsatisfactory	Bare minimum	Satisfactory effort & performance	Above & Beyond
Alternative Design	0: No redesign or irrelevant redesign.	1: The student's redesign was not significantly different from the original design and seems unlikely to satisfy a real user need.	3: The student's redesign is significantly different from the original design, but seems unlikely to satisfy a real user need.	5: The student's redesign is significantly different from the original design and seems to satisfy a real user need.
Extra Credit: Electronic Prototype of Redesign	0: No URL to interactive prototype.	1: The prototype is incomplete and barely interactive.	3: The prototype is somewhat interactive, but not ready for user testing.	5: The alternative prototype is fully interactive and ready for user testing.
User Tests	0: No user tests were performed.	1: The user tests did not capture much useful information and were not carried out in a serious, planned way.	3: The user tests captured some information, but are incomplete in some way.	5: The user tests captured a great deal of information that would result in important interface changes.
Photos/Sketches	0: No photographs were submitted that showed interesting moments.	1: 1 photograph was submitted that showed an interesting moment.	3: 2 photographs were submitted that showed interesting moments.	5: At least 3 photographs were submitted, and all three photographs showed interesting moments in the user testing process.

Results Summary	0: No user testing results summary provided.	1: The summary suggests the user tests were not done well enough to reveal any problems in the interface.	3: The summary suggests the user tests found some problems in the interface, but the problems could have been discovered without user tests.	5: The summary suggests the user tests revealed significant insights into the design that could trigger positive changes for the interface.
Changes	0: No changes listed or irrelevant changes.	1: No useful changes to the interface resulted from the user tests.	3: The student presented several possible changes derived from the user testing data, although not all of the changes were useful or some important changes were overlooked.	5: The student suggested several possible changes based on the user testing, all of which were important and directly addressed the problems identified in user testing.